

### The mass equation law for binding of a protein P to its DNA

$$D_{\text{free}} + P_{\text{free}} \stackrel{\Longrightarrow}{\longleftarrow} DP$$
  $K_1 = \frac{D_{\text{free}} \cdot P_{\text{free}}}{DP}$ 

binding of the first proteins with the dissociation constant  $K_1$ 

 $D_{\text{free}}$ , concentration free DNA;  $P_{\text{free}}$ , concentration free protein

binding constant 
$$K_{\rm B} = \frac{1}{\text{dissociation constant } K_{\rm D}}$$

### How fast is binding or dissociation

$$AB \xrightarrow{k_{off}} A+B$$

k<sub>off</sub> in s<sup>-1</sup> is the reaction rate constant for dissociation

 $k_{on}$  in  $M^{-1}$  s<sup>-1</sup> is the reaction rate constant for binding

$$\frac{k_{\text{off}}}{k_{\text{on}}} = K_{\text{d}}$$

relation to the equilibrium dissociation constant

$$\frac{1}{k_{\text{off}}} = \tau$$

life time of the complex

$$\frac{d[AB]}{dt} = k_{on} \cdot [A] \cdot [B] - k_{off} \cdot [AB]$$

rate equation for complex formation, can be solved but it is already difficult

k<sub>on</sub> cannot be higher than 10<sup>8</sup> - 10<sup>9</sup> M<sup>-1</sup> s<sup>-1</sup> for a diffusion controlled reaction

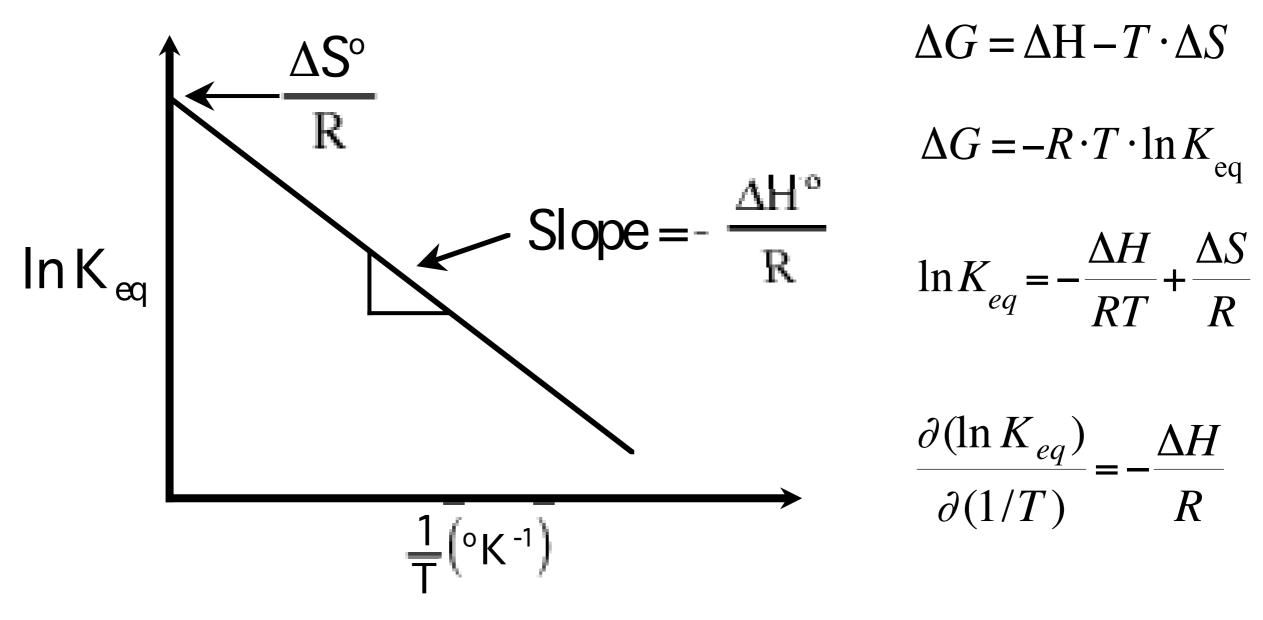
# What is the meaning of the dissociation constant for binding of a single ligand to its site?

- 1.  $K_d$  is a concentration and has units of mol per liter
- 2.  $K_{\rm d}$  gives the concentration of ligand that saturates 50% of the sites (when the total sit concentration is much lower than  $K_{\rm D}$ )
- 3. Almost all binding sites are saturated if the ligand concentration is 10 x  $K_d$
- 4. The dissociation constant  $K_d$  is related to Gibbs free energy  $\Delta G$  by the relation  $\Delta G = -R T \ln(K_d)$

### Our energy and time coordinate system

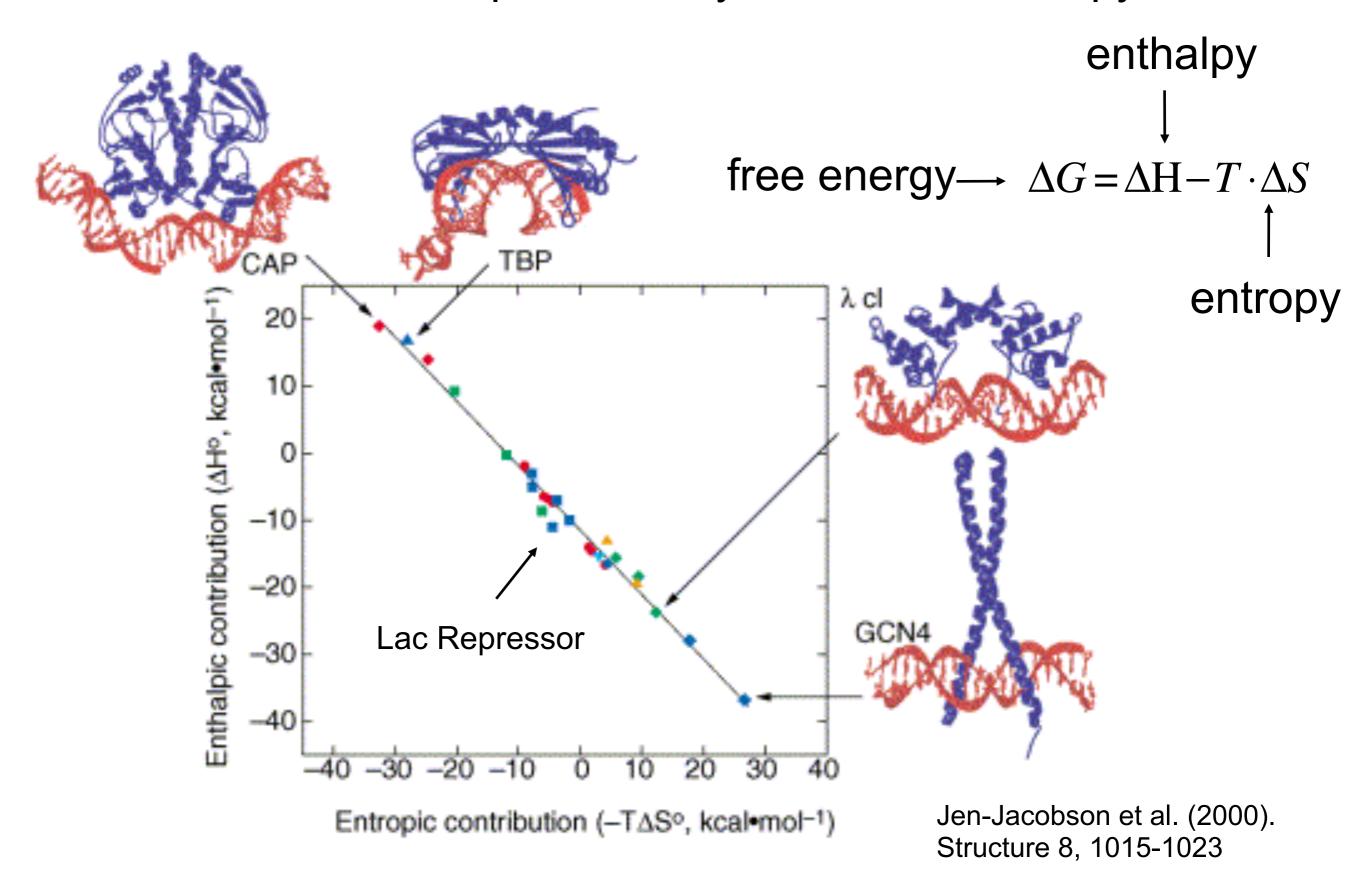
K <sub>d</sub> (M)	concentration scale	$\Delta G$ $k_{off}$ (kcal/mol) (s-1)		complex life time	Binding interaction		
			for $k_{on} = 10^5 M^{-1} s^{-1}$				
10-3	1 mM	-4.1	10 <sup>2</sup>	10 ms	ion-DNA ion-protein		
10-4	0.1 mM	-5.5	10 ¹	0.1 sec			
10 <sup>-5</sup>	10 μΜ	-6.8	1	1 sec	enzyme-ligand (weak)		
10-6	1 uM	-8.2	10-1	10 sec	protein-DNA, unspecific		
10-7	0.1 μΜ	-9.5	10-2	100 sec	enzyme-ligand (strong)		
10-8	10 nM	-10.9	10-3	16.7 min			
10 <sup>-9</sup>	1 nM	-12.3	10-4	2.8 hours	protein-DNA specific		
10-10	0.1 nM	-13.6	10 <sup>-5</sup>	28 hours			
10-11	10 pM	-15	10-6	11.6 days	antibody-antigen		
10-12	1 pM	-16.4	10 <sup>-7</sup>	116 days			

# The temperature dependence of the binding constants reveals $\Delta H$ and $\Delta S$ in a van't Hoff plot if $\Delta H$ and $\Delta S$ are independent of temperature



From the slope of ln  $K_{eq}$  vs. 1/T (usually from 0 to 40 °C) one can determine the  $\Delta H$  and from extrapolation also  $\Delta S$ . Is the van't Hoff plot curved then  $\Delta H$  is temperature dependent and it can be determined from the derivative.

# The unfavorable enthalpy contribution associated with DNA distortion is compensated by a favorable entropy



### K<sub>D</sub> and ∆G values for protein-DNA binding per site

Specific binding of a protein to DNA varies over a relatively small range of  $\Delta G_{bind.sp}$  = -9 to -16 kcal/mol, with ~60 kcal/mol for  $\Delta H$  and  $\Delta G_{bind.sp}$ 

$$\Rightarrow \Delta G_{bind,sp} \approx const. (-11.7 \pm 1.6 kcal/mol)$$

$$\Rightarrow \Delta H = -T \cdot \Delta S - 11.7 \text{ kcal/mol}$$

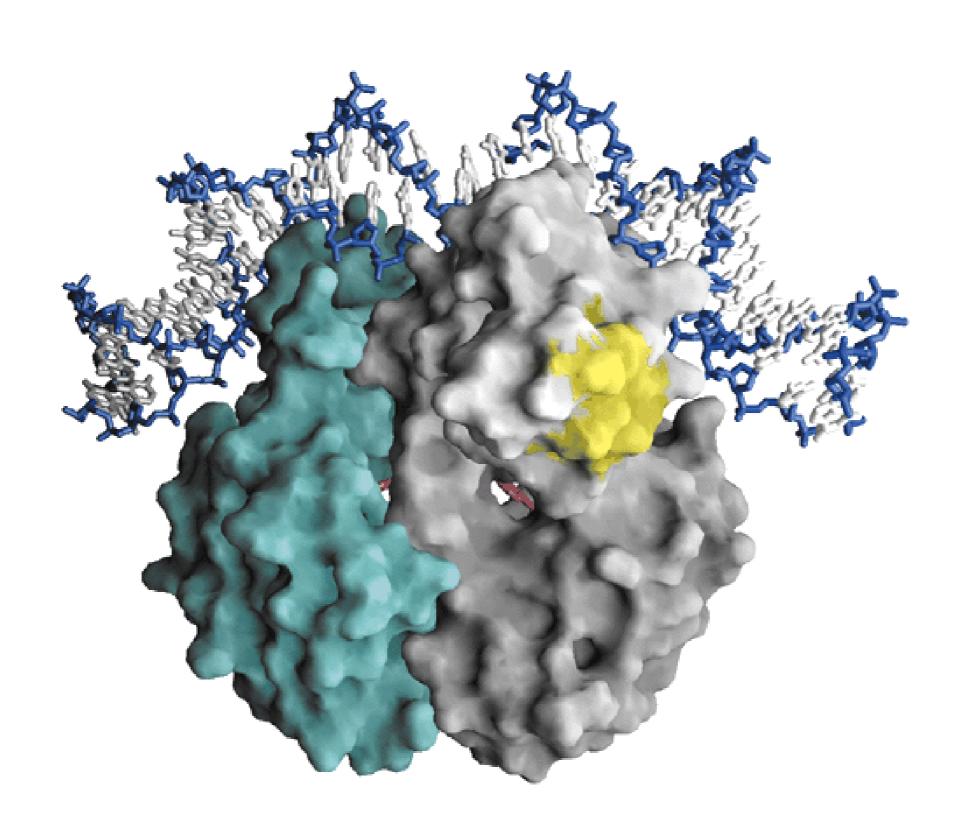
Protein needs to select specific binding site from unspecific sites

$$\Rightarrow$$
  $\Delta\Delta$ G(specific - unspecific)  $\sim$  -5 to -9 kcal/mol

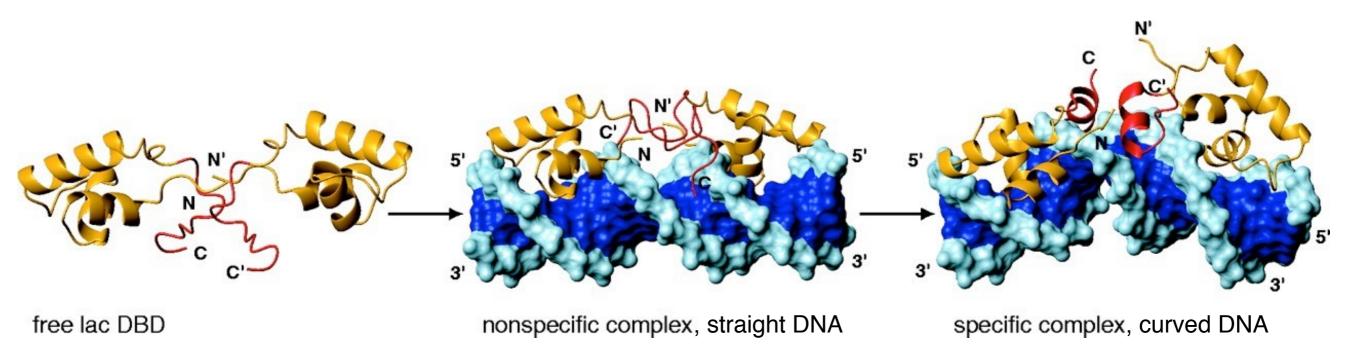
Protein binding must be reversible on the cell's time scale

$$\Rightarrow \Delta G_{bind,sp} \leq -16 \text{ kcal/mol}$$

### Molecular structure of E. coli CRP (also called CAP for catabolite gene activator protein)

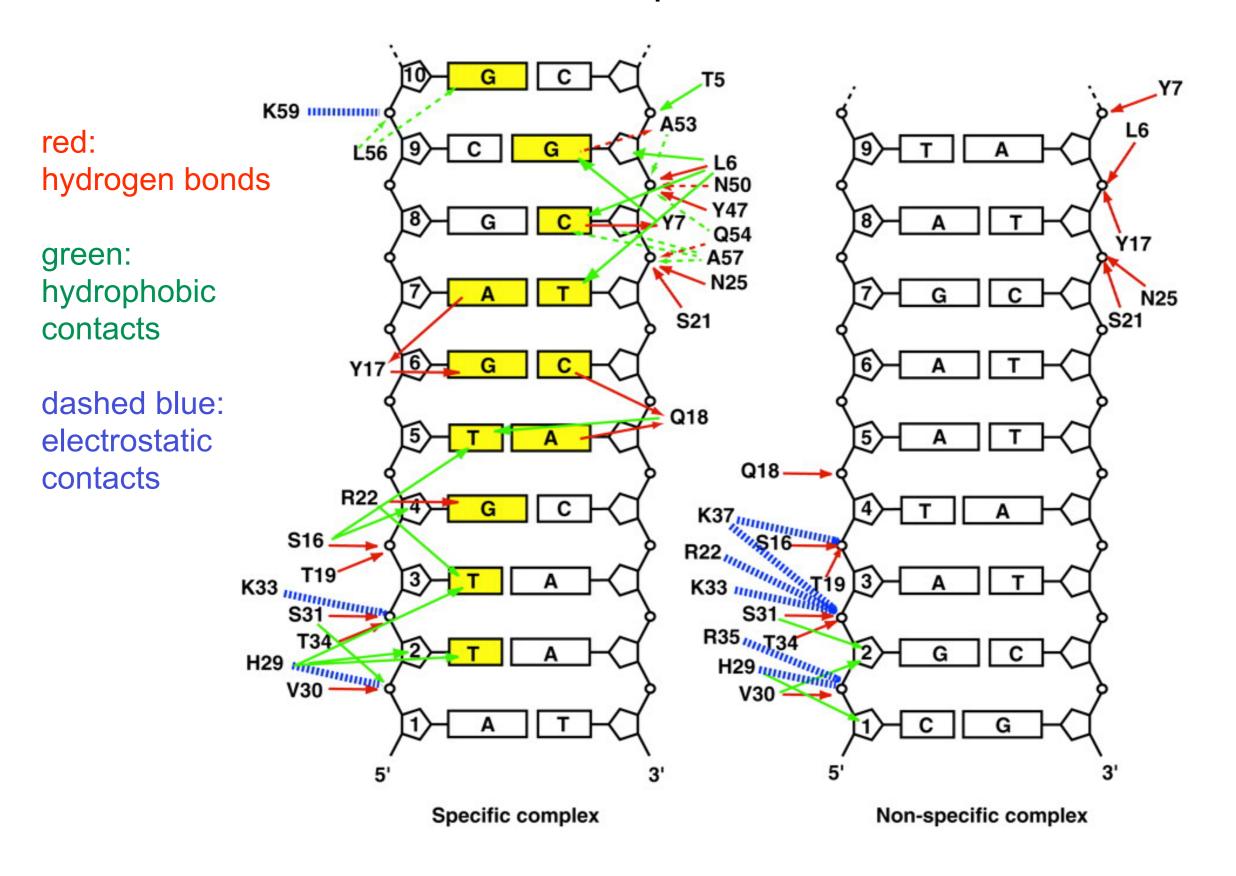


### The hinge region (50-62 in red) of Lac-DBD is folded only in the specific complex with DNA

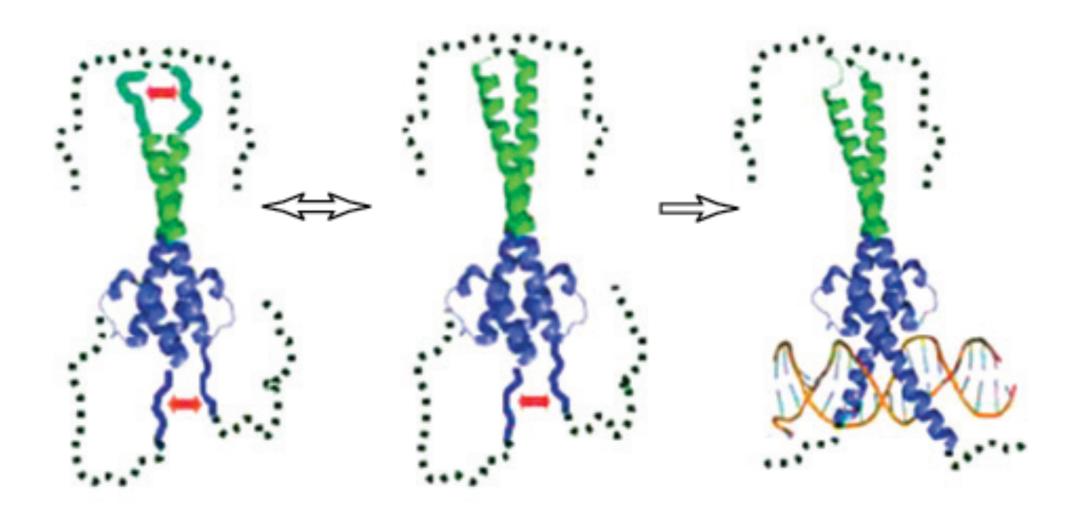


- folding of hinge region with specific contacts in minor groove
- specific interactions major groove
- less electrostatic interactions
- curvature of DNA

### Specific (left) and nonspecific (right) protein-DNA contacts of Lac-DBD repressor with DNA

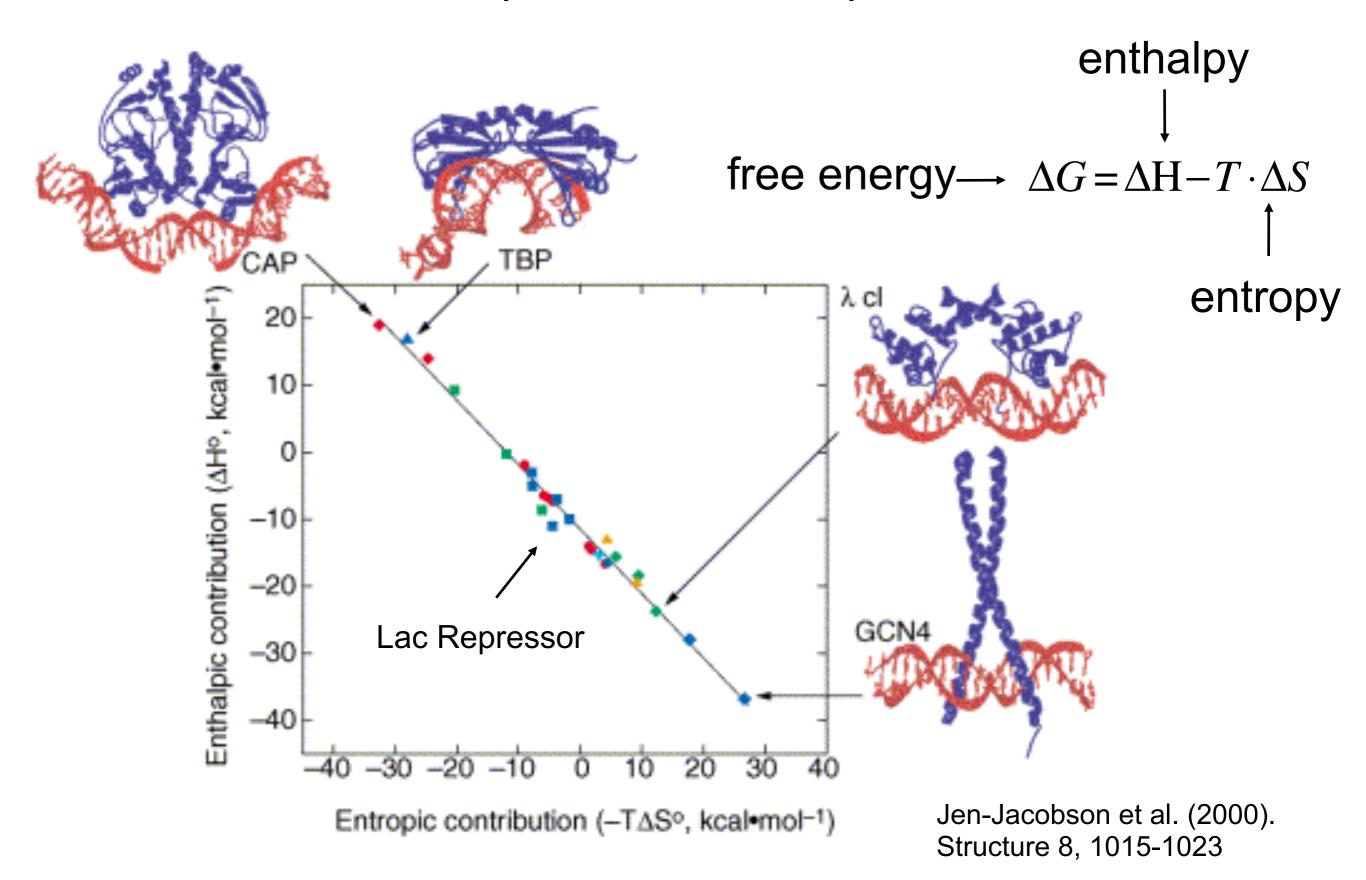


# Local folding of the Max transcription factor upon dimerization and binding



The Max transcription factor (PDBcode: 1NKP) binds DNA as a dimer. The disordered N-terminal region (upper dotted line) reduces the electrostatic repulsion (red arrows) between the two monomers, and increases the population of the folded state at the flanking leucine zipper (green). This also stabilizes the bHLH region (blue) and thus improves binding affinity for DNA.

## Contribution of enthalpy and entropy to binding energies of different protein-DNA complexes



#### Coupling of Local Folding to Site-Specific Binding of Proteins to DNA

Ruth S. Spolar and M. Thomas Record Jr.

Science 263, 777-784, 1994

The analysis of  $\Delta S_{,i}$  for protein binding to DNA is conducted at the characteristic temperature  $T_{s}$  where  $\Delta S_{bin} = 0$  so that:

$$\Delta S_{bin}(T_S) = 0 = \Delta S_{HE}(T_S) + \Delta S_{rt} + \Delta S_{PE} + \Delta S_{other}$$

The term  $\Delta S_{other}$  arises primarily from folding/conformational changes in the protein and/or the DNA upon specific DNA binding.

Protein folding includes two dominant and opposing contributions to the entropy:

- a) One positive from the hydrophobic effect or the "release" of water on burial of nonpolar surfaces
- b) One negative from the reduction in conformational entropy

$$\Delta S_{fold}(T_S) = 0 = \Delta S_{HE}(T_S) + \Delta S_{conf}$$

### S&R, Table 1: Protein folding

 $325 \text{ cal } \text{K}^{-1}\text{mol}^{-1}/56 \text{ residues}$ = 5.8 cal K<sup>-1</sup>mol<sup>-1</sup> :

Table 1. Entropic contribution	n to	protein	folding	from	the	hydrophobic	effect.
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Protein	R	$-\Delta C_{ m fold}^{ m o}$ (cal mol $^{-1}$ K $^{-1}$ )	7 <sub>S</sub> * (K)	$-\Delta A_{np}^{\dagger}$ $(\mathring{A}^2)$	$\Delta S_{\rm HE}^{\circ}(T_{\rm S})\ddagger$ (e.u.)	$-\Delta s_{\text{other}}^{\circ} $ (e.u.)
Streptococcal protein G, domain B1	56	620 (68)	272	2900	325	5.8
BPTI	58	720 (24)	306	2640	196	3.4
		400 (69)	221		471	8.1
Parvalbumin b	108	1100 (70)	268	5485	640	5.9
Ribonuclease A	124	1230 (25)	255	5815	771	6.2
Lysozyme (hen egg white)	129	1540 ( <i>25</i> )	270	6870	786	6.1
Ferricytochrome c	104	1730 ( <i>25</i> )	294	5540	483	4.6
Staphylococcal nuclease	141	1820 ( <i>25</i> )	288	7880	738	5.2
Holo <sup>II</sup> myoglobin	153	2770 ( <i>25</i> )	301	9710	773	5.1
β trypsin	223	2850 ( <i>25</i> )	281	11830	1200	5.4
Papain	212	2920 ( <i>25</i> )	290	12755	1167	5.5
α chymotrypsin	245	3020 ( <i>25</i> )	280	14770	1517	6.2
Carbonic anhydrase	256	3820 ( <i>25</i> )	290	15760	1442	5.6
Pepsinogen	370	6090 (25)	297	23730	1990	5.4
water saccisable nonpolar	rd-kegr	mos ambitos stari	patien.	us demonu	Average	9 5.6 ± 0.5

<sup>\*</sup>Values of  $T_{\rm S}$  were calculated from values of  $\Delta C_{\rm fold}^{\rm o}$  and  $\Delta S_{\rm fold}^{\rm o}$  cited in the reference indicated in column 3. Reported uncertainties in  $\Delta C_{\rm fold}^{\rm o}$  range from 5 to 20 percent. Corresponding uncertainties in  $T_{\rm S}$  range from 1 to 7 K degrees. †Calculations of  $\Delta A_{\rm np}$  model the denatured state as an extended  $\beta$  chain (26, 27). The value of  $\Delta A_{\rm np}$  for folding the B1 domain of streptococcal protein G was calculated as described in (27) from Brookhaven Protein Database (67) file 2GB1. All other values of  $\Delta A_{\rm np}$  are from (26). ‡Eq. 2.  $\Delta S_{\rm other}^{\rm o} = \Delta S_{\rm other}^{\rm o}/\Re$ , calculated from Eq. 4. |In this and subsequent tables, holo refers to the protein associated with its cofactor. Not including BPTI.

 $T\Delta S_{conf} = 5.6 \text{ cal K}^{-1} \text{ mol}^{-1} \cdot 298 \text{ K} = 1.7 \text{ kcal mol}^{-1}$ 

Conclusion 1 from entropy analysis:

The unfavorable conformational entropy of folding per residue is

 $\Delta$ S = -5.6 cal mol<sup>-1</sup> K<sup>-1</sup> or T $\Delta$ S = -1.7 kcal mol<sup>-1</sup>

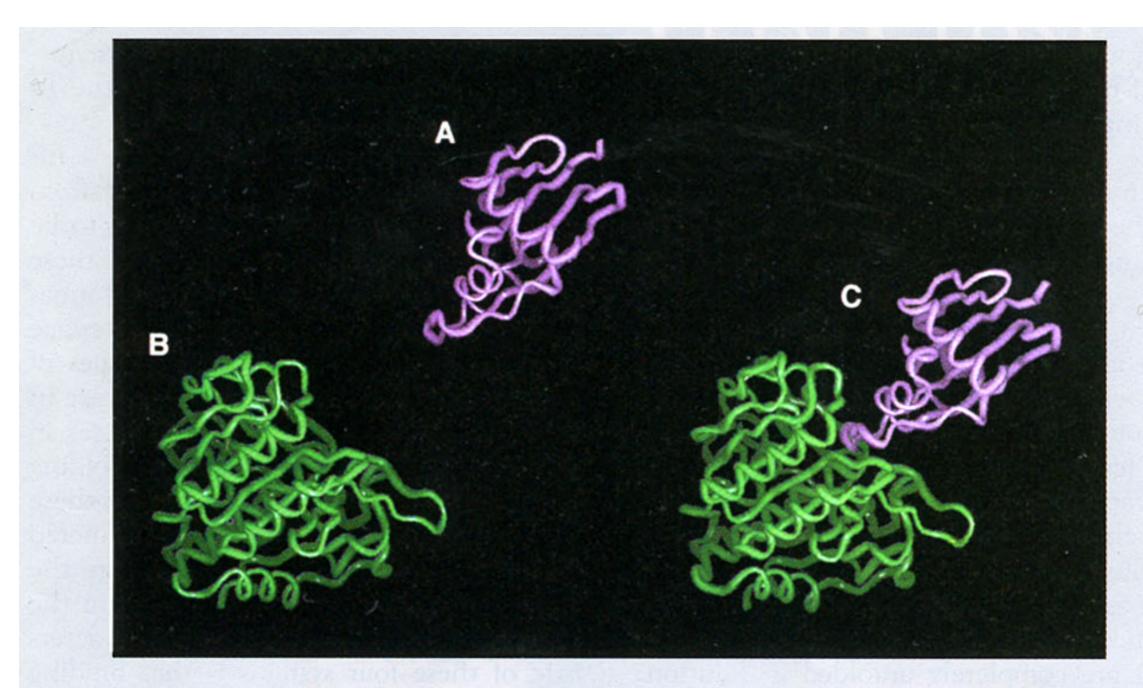
# $\Delta S_{rt}$ : Unfavorable entropy due to loss of movements of protein upon binding

Loss of rigid body rotational and translational entropy  $\Delta S_{rt}$ 

$$\Delta S_{bin}(T_S) = 0 = \Delta S_{HE}(T_S) + \Delta S_{rt}$$

Estimated from studies of entropic changes arising from rigid body protein-protein association.

# S&R, Fig. 1: Rigid body association for subtilisin binding to its inhibitor protein



**Fig. 1.** Ribbon model of a "rigid-body" association. X-ray crystallographic structures of (**A**) subtilisin inhibitor monomer (purple, PDB file 2SSI) and (**B**) uncomplexed subtilisin (green, PDB file 2ST1), shown in the same orientation as in the complex. (**C**) Enzyme-inhibitor complex (PDB file 2SIC), same colors as in (A).

### S&R, Table 3: Rigid body association

Table 3. Entropic contributions to "rigid body" associations.

Process	$-\Delta C_{\rm assoc}^{\rm o}$ (cal mol $^{-1}$ K $^{-1}$ )	<i>T</i> <sub>s</sub> * (K)	$\Delta S_{\text{HE}}^{\circ}(T_{\text{S}})^{\dagger}$ (e.u.)
Soybean inhibitor + trypsin → complex	440 (83)	349	(60)
Subtilisin inhibitor + subtilisin monomer → complex	240 (71)	339	`41´ ~
Subtilisin inhibitor + α chymotrypsin monorner → complex	270 (84)	343	(43)
FK506 + FKBP-12 → complex	260 (73)	289	60
aris tive to I's and to the molecular mess of the	sydeny, suriger are	Avera	ge: 50 ± 10

<sup>\*</sup>Values of  $T_{\rm S}$  calculated from values of  $\Delta C_{\rm assoc}^{\rm o}$ ,  $\Delta S_{\rm assoc}^{\rm o}$  in references cited in column 2. † $\Delta S_{\rm HE}^{\rm o}(T_{\rm S})$  calculated from Eq. 2 with values for  $\Delta A_{\rm np}$  from Table 2. Values of  $\Delta S_{\rm HE}^{\rm o}(T_{\rm S})$  in parentheses are calculated from Eq. 3 for systems lacking structural data to evaluate  $\Delta A_{\rm np}$ .

$$\Delta S_{bin}(T_S) = 0 = \Delta S_{HE}(T_S) + \Delta S_{rt}$$

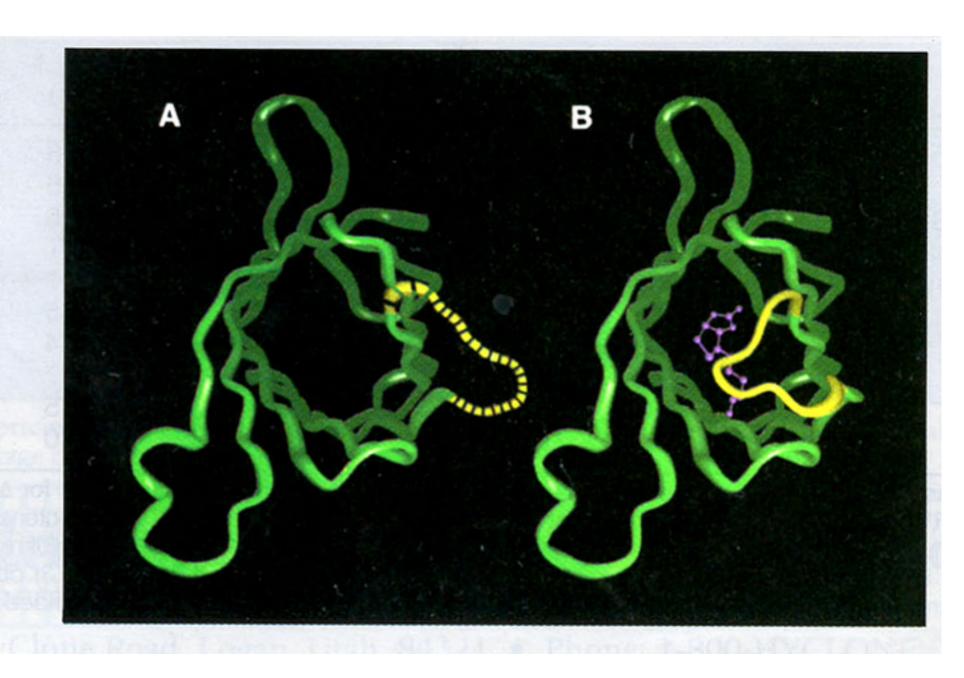
Conclusion 2 from entropy analysis:

The unfavorable entropy for rigid body association of two macromolecules is

 $\Delta S = -50 \text{ cal mol}^{-1} \text{ K}^{-1}$  or  $T\Delta S = -14.9 \text{ kcal mol}^{-1}$ 

# S&R, Fig. 2: Induced folding of an avidin monomer upon binding to biotin

Fig. 2. Ribbon model of avidin-biotin "induced fit" interaction. (A) Model of the uncomplexed avidin monomer in solution (green). Residues (36-44) (dashed loop in yellow) are disordered in the free crystal structure (49) and are inferred to be in a flexible coil state of high conformational entropy in solution. (B) Avidin-biotin complex. Ordering of the looped region (yellow) upon binding encloses biotin (in purple) in a "hydrophobic box" (49).



#### S&R, Table 4: Coupled folding in protein-protein association

R (number of residues involved in folding transition)

=  $\Delta S_{\text{other}}$  /-5.6 cal K<sup>-1</sup> mol<sup>-1</sup>

Table 4. Entropic contributions where folding is coupled to ass	sociation, predictions of the number	of residues participating	in the folding transition
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Process (structural references)	T <sub>S</sub> * (K)	$\Delta S_{\rm HE}^{\circ}(T_{\rm S})^{\dagger}$ (e.u.)	$\Delta S_{\rm rt}^{\rm o}$ ‡ (e.u.)	$\Delta S_{ ext{other}}^{\circ} $ (e.u.)	Mth	₩str¶
Angiotensin II (48) + antibody Fab 131 (85) → complex (85)	312	68	-50	<del>-18</del>	3	8
Avidin (49) + biotin → complex (49)	291	85	-50	-35	6	9
S-peptide (47) + S-protein (47) → ribonuclease S (86)	253#	145	-50	-95	17	15
L-tryptophan + apo Trp R monomer (11) → complex (12)	263	127	-50	-77	14	17**
Holo Trp R dimer (11) + trp operator DNA $\rightarrow$ complex (12)	319	147	-50	-97	17	16
2 GR DBD (13) + DNA → complex (14)	308	285	-100	-185	33	40
3 glucagon (81) $\rightarrow$ trimer (87)	271	364	-100	-264	47	48_72
4 melittin (82) → tetramer (88)	313	477	-150	-327	58	104
2 arc repressor (78) → dimer (77)	289	525	-50	-475	85	80-92
$2 \lambda \text{ cro repressor } (80) \rightarrow \text{dimer } (79)$	287	620	-50	-570	102	120

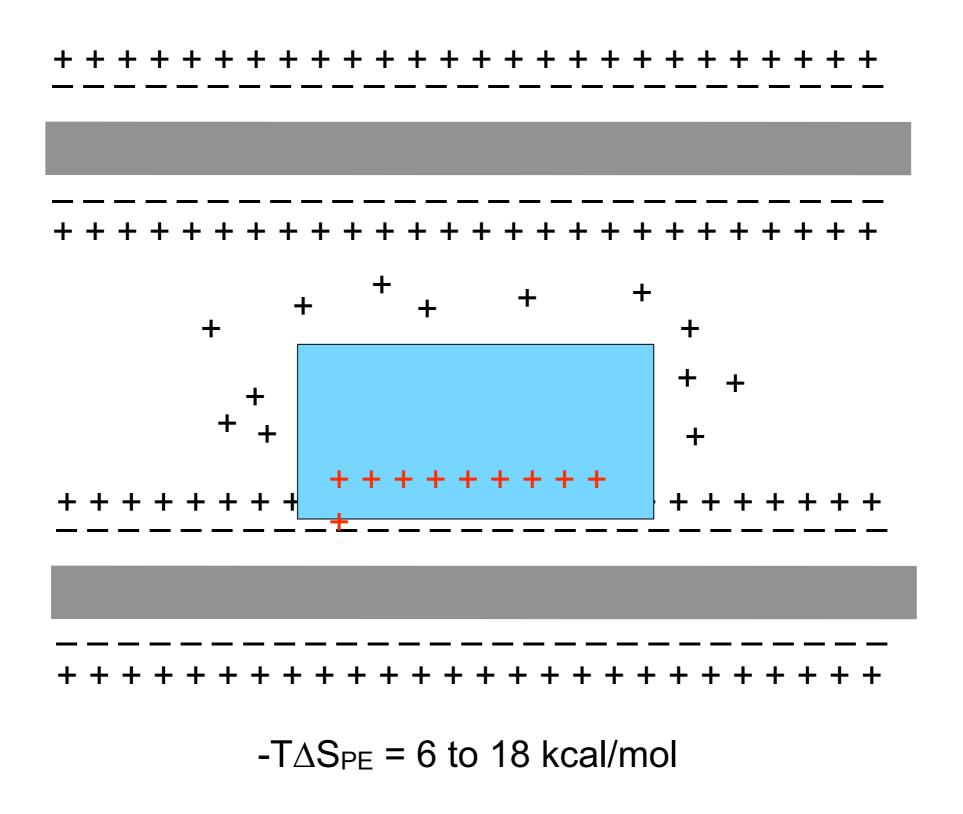
\*References for data used to calculate  $T_{\rm S}$  are the same as those for  $\Delta C_{\rm assoc}^{\rm o}$  in Table 2. † $\Delta S_{\rm HE}^{\rm o}(T_{\rm S})$  evaluated from Eq. 2 with values for  $\Delta A_{\rm np}$  from Table 2. ‡Table 3. §Eq. 5. ||Eq. 6. Propagated uncertainties in  $\Re^{\rm th}$  increase from ±15 percent for  $\lambda$  cro repressor to ±50 percent for angiotensin II, and are typically ±25 percent. ¶ $\Re^{\rm str}$  represents the difference between the number of residues folded in the crystal structure of the complex and the number of residues observed to be folded in the free species by NMR, x-ray, or CD as referenced in column 1. # $T_{\rm S}$  estimated from values of  $\Delta C_{\rm assoc}^{\rm o}$  (273) and  $\Delta S_{\rm assoc}^{\rm o}$  (273) obtained from the temperature dependence of  $\Delta C_{\rm assoc}^{\rm o}$  given in (47), based on the assumption that S protein is completely native at 273 K. \*\*Number of residues folded in the complex based on the NMR structure.

$$\Delta S_{bin}(T_S) = 0 = \Delta S_{HE}(T_S) + \Delta S_{rt} + \Delta S_{other}$$

### Conclusion 3 from entropy analysis:

The number of residues involved in the folding transition can be calculated from the  $\Delta S_{\text{other}}$  term and the value of  $\Delta S = -5.6$  cal mol<sup>-1</sup> K<sup>-1</sup> derived from the entropy analysis of protein folding.

#### $\Delta S_{PE}$ : Favorable displacement of ions from the DNA



#### Summary of protein and DNA energy contributions

- very different distributions of  $\Delta H$  and  $\Delta S$  for protein-DNA binding but similar  $\Delta Gs$
- Unfavorable conformational entropy of folding per residue or 1.7 kcal mol<sup>-1</sup>
- can be compensated by the hydrophobic effect or the "release" of water on burial of nonpolar surfaces
- for rigid body association 15 kcal mol<sup>-1</sup> rotational/translation entropy loss
- additional folding of the protein can occur during binding to very different degrees
- 6-18 kcal mol<sup>-1</sup> depending on interaction surface or the displacements of counter-ions upon protein binding to DNA which drives binding